

## A Sky Radiation-Illumination Correlation Model

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### ABSTRACT

A radiation-illumination correlation model was tested successfully for accuracy against empirical data collected by the Daylighting Laboratory of the Solar Energy Research Institute (SERI).

The model developed proposes the possibility of a single measured value (e.g., total solar radiation) being sufficient to calculate the direct and diffuse components of radiation as well as the illumination of a surface of arbitrary orientation.

The sky radiation model was developed based on an empirical correlation between the two Liu-Jordan parameters. The model automatically weights the contribution from the direct solar beam and from the diffuse visible sky radiation, without arbitrary assignment of the clear or overcast labels.

The illumination on a surface was calculated by multiplying the intercepted radiation by the total luminous efficacy of the sky, obtained from the empirical measurements reported by SERI.

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## INTRODUCTION

The work summarized in this paper is part of a continuing effort to develop validated algorithms for the Dynamic Energy Response of Buildings (DEROB) system of simulation computer programs. The paper presents an algorithm based on phenomenological relationships which allows the calculation of the illumination, due to the presence of the sky, on an unobstructed plane. The phenomenological relationship is based on analysis of radiation and illumination data collected by the Daylighting Laboratory of SERI<sup>1</sup>.

There are three key assumptions in the algorithm: (1) the total illumination on a plane can be obtained by multiplying the total radiation on the plane by the total luminous efficacy of the sky; (2) the total radiation is the arithmetic sum of the direct and diffuse radiation, with each of these components multiplied by a weighting factor (determined by the total atmospheric extinction factor); and (3) the total atmospheric extinction factor and the two weighting factors can be calculated from a single empirical measurement, such as the total radiation on the horizontal surface.

Throughout the five sections of the paper the SERI measurements are compared with the values calculated by this algorithm. The sections include (1) the step by step description of the calculation sequence; (2) the rationale behind the use of the weighting factors mentioned previously; (3) the rationale for the use of the total luminous efficacy; (4) the derivation of the orientation factors (needed for the calculation of illumination on surfaces of arbitrary orientation); from sky luminance; and (5) a summary of the SERI data made available to this author.

## ILLUMINATION CALCULATION SEQUENCE

This section describes the illumination calculation sequence in detail. The sequence starts with the specifications of the empirical data required, follows with the intermediate calculations, and concludes with the illumination values. A running example is included in the description.

### Empirical Data

The empirical data include the data needed to locate the position of the sun in the sky and the data required to describe the sky conditions.

Solar Position. The solar position in the sky is defined by specifying the latitude angle,  $L$  (northern latitudes are positive angles, southern latitudes are negative angles); the day of the year,  $D$ ; and the hour of the day,  $h$ , measured in solar time.

Sky Condition. The sky condition is described by specifying the total solar radiation,  $T$ , measured on the horizontal plane; it is also possible to use diffuse or direct radiation measurements. However, the most straightforward calculation is obtained with the total radiation on the horizontal plane. Most of the solar radiation data available in weather tapes include this value; consequently, this value is the most convenient for the purpose of this calculation.

Example. The example to be used throughout this calculation sequence uses data taken from the SERI measurements: Latitude,  $L = 40^{\circ}\text{N}$ ; Julian day,  $D = 68$ ; time of day,  $h = 11.2$ . Total Solar Radiation on the horizontal plane,  $T = 237.75 \text{ BTU/hr ft}^2$  ( $750 \text{ W/m}^2$ ).

### Solar Position Calculation

The solar position is described with the direction cosines. They are calculated from the hour angle, the solar declination angle, and the latitudinal angle. The hour angle is given by

$$H = 15(n - 6) \quad (1)$$

The solar declination angle by  

$$\delta = 23.5 \cos [360(D - 172)/365] \quad (2)$$

The Direction Cosines. The direction cosines are given by

$$\sigma_s = \sin(L) \sin(H) \cos(\delta) - \cos(L) \sin(\delta) \quad (3)$$

$$\sigma_e = \cos(\delta) \cos(H) \quad (4)$$

$$\sigma_z = \cos(L) \sin(H) \cos(\delta) + \sin(L) \sin(\delta) \quad (5)$$

for the south, east, and zenith directions, respectively.

Example.  $H = 75^\circ$   
 $\delta = -5.1^\circ$   
 $\sigma_s = 0.69$   
 $\sigma_e = 0.26$   
 $\sigma_z = 0.68$

Sky Condition Calculation

The sky condition is described by the dimensionless ratio of the measured total radiation to the atmosphere free radiation.

The atmosphere free radiation is given by

$$S = \sigma_z S_c \quad (6)$$

where

$S_c = 428.9 \text{ BTU/hr ft}^2 \text{ (1353 W/m}^2\text{)}$  is the solar constant

The Total Extinction Factor. The ratio of the measured total radiation to the atmosphere free value is called the total extinction factor, and it is given by

$$x = T/S \quad (7)$$

The Direct Extinction Factor. The ratio of direct solar radiation on the horizontal plane to the atmosphere free value is the direct extinction factor.

$$\alpha = x^{n+1} \quad (8)$$

$$n = 3.37[1 - e^{-(1-x)}] \quad (9)$$

Therefore, the direct radiation incident on the horizontal plane is given by

$$D = \alpha S \quad (10)$$

The Diffuse Extinction Factor. This model assumes that the total radiation is the arithmetic sum of the direct and diffuse values. The diffuse radiation is given, therefore, by

$$d = (x - \alpha) S \quad (11)$$

The difference,  $(x - \alpha)$ , can be defined as the diffuse extinction factor.

Example.  $S = 291 \text{ BTU/hr ft}^2 \text{ (920 W/m}^2\text{)}$   
 $x = 0.82$   
 $n = 0.56$   
 $D = 214.29 \text{ BTU/hr ft}^2 \text{ (676 W/m}^2\text{)}$   
 $d = 25.04 \text{ BTU/hr ft}^2 \text{ (79 W/m}^2\text{)}$

Figure 1 shows the radiation values on the horizontal plane, as measured by SERI, and the values calculated using Eq 11 for the diffuse component.

### The Maximum Direct Radiation and Illuminance

The maximum direct radiation on a flat surface is given by

$$D_{\max} = \alpha S_c \quad (12)$$

The maximum direct illuminance is obtained by multiplying the maximum direct radiation by the sky luminous efficacy

$$e = 33.4 \text{ lm h/BTU (114 lm/W)} \quad (13)$$

Example.  $D_{\max} = 319 \text{ BTU/hr ft}^2 \text{ (1008 W/m}^2\text{)}$

$$\text{Maximum Direct Illuminance} = 10.2 \times 10^3 \text{ foot candles (11} \times 10^4 \text{ lx)}$$

### The Maximum Diffuse Radiation and Illuminance

The maximum diffuse radiation is given by

$$d_{\max} = (x - \alpha) \sigma_z S_c \quad (14)$$

and the maximum diffuse illuminance is the product of the maximum diffuse radiation when multiplied by the sky luminous efficacy.

Example.  $d_{\max} = 25.04 \text{ BTU/hr ft}^2 \text{ (79 W/m}^2\text{)}$

$$\text{Maximum Diffuse Illuminance} = 836 \text{ ft candle (.9} \times 10^4 \text{ lx)}$$

### The Orientation Dependence

The maximum direct radiation occurs in the area where the solar disk is located, whereas the maximum diffuse radiation occurs in the zenith direction. To obtain the illuminance on a plane of arbitrary orientation, the unit vector normal, as defined by its zenith angle,  $\theta$ , and its azimuth angle,  $\phi$ , measured from the south axis, must first be calculated. The components of this unit vector normal are given by

$$n_s = \sin \theta \cos \phi \quad (15)$$

$$n_e = \sin \theta \sin \phi \quad (16)$$

$$n_z = \cos \theta \quad (17)$$

The Direct Component. The orientation factor for the direct component of the illuminance is found projecting the solar position vector onto the vector normal to the surface:

$$\sigma_n = \sigma_s n_s + \sigma_e n_e + \sigma_z n_z \quad (18)$$

The direct radiation is the product of the maximum direct radiation times the orientation factor

$$D(\hat{n}) = D_{\max} \sigma_n \quad (19)$$

Note that whenever the value of  $\sigma_n$  is negative the surface is facing away from the sun and the value of  $\sigma_n$  should be set to zero.

The Diffuse Component. The orientation factor for the diffuse component is given by the following factor;

$$M = \frac{3}{7} \left[ (1 + \cos \theta) / 2 + \frac{4}{3} \left( 1 - \frac{\theta}{\pi} \right) \cos \theta + \frac{4 \sin \theta}{3 \pi} \right] \quad (20)$$

This expression is obtained by integrating the Moon Spencer luminance expression for an overcast sky over the visible sky dome.

The diffuse radiation is obtained by multiplying the maximum diffuse radiation times the orientation factor, M:

$$d(\hat{n}) = d_{\max} M \quad (21)$$

Note that M depends only on the zenith angle.

Total Radiation. The total radiation on a surface of arbitrary orientation is expressed by

$$T(\hat{n}) = D(\hat{n}) + d(\hat{n}) \quad (22)$$

Total Illuminance. The total illuminance is the product of the total radiation times the sky luminous efficacy:

$$E = \epsilon T(\hat{n}) \quad (23)$$

Example. Consider three vertical planes in addition to the horizontal plane:

a. vertical east:  $\theta = 90, \phi = 90$

$$n_s = 0, n_e = 1, n_z = 0$$

$$\sigma_n = 0.26$$

$$M = 0.40$$

$$T(\hat{n}) = 92 \text{ BTU/hr ft}^2 \text{ (291 W/m}^2\text{)}$$

$$E = 3.08 \times 10^3 \text{ ft. candle (3.31} \times 10^4 \text{ lx)}$$

b. vertical south:  $\theta = 90, \phi = 0$

$$n_s = 1, n_e = 0, n_z = 0$$

$$\sigma_n = 0.69$$

$$M = 0.40$$

$$T(\hat{n}) = 224.43 \text{ BTU/hr ft}^2 \text{ (708 W/m}^2\text{)}$$

$$E = 7.50 \times 10^3 \text{ ft. candle (8.07} \times 10^4 \text{ lx)}$$

c. vertical west:  $\theta = 90, \phi = -90$

$$n_s = 0, n_e = -1, n_z = 0$$

$$\sigma_n = -0.26 \text{ since } \sigma_n < 0 \text{ } \sigma_n = 0$$

$$M = 0.40$$

$$T(\hat{n}) = 12.36 \text{ BTU/hr ft}^2 \text{ (39 W/m}^2\text{)}$$

$$E = 418.06 \text{ ft. candle (0.45} \times 10^4 \text{ lx)}$$

d. horizontal plane:  $\theta = 0$

$$n_s = 0, n_e = 0, n_z = 1$$

$$\sigma_n = 0.68$$

$$M = 1$$

$$T(\hat{n}) = 237.75 \text{ BTU/hr ft}^2 \text{ (750 W/m}^2\text{)}$$

$$E = 7.94 \times 10^3 \text{ ft. candles (8.55} \times 10^4 \text{ lx)}$$

Figures 2 and 3 show the SERI illumination measurements along each of the four cardinal coordinates and compare them with the values calculated with Eq 23. Figure 2 shows the results for the overcast day and Fig. 3 for the clear day.

Ground Reflections. Assuming that the ground is an infinite horizontal plane capable of only diffuse reflections, the illumination on a surface due to ground reflection is give by

$$R = r \left( \frac{1 - \cos\theta}{2} \right) \sigma_z S_c x \quad (24)$$

where

r is the reflectance coefficient of the ground

Example. If the reflectance coefficient, r, equals 0.06, the illumination due to the ground is

(a) horizontal plane facing up-- $\theta = 0, R = 0$

(b) vertical plane-- $\theta = 90, R = 242 \text{ ft candles (.26} \times 10^4 \text{ lx)}$

(c) horizontal plane facing down-- $\theta = 180, R = 483 \text{ ft. candles (0.52} \times 10^4 \text{ lx)}$

## SOLAR RADIATION

The diffuse total solar radiation are often displayed in terms of the dimensionless ratios<sup>2</sup>.

$$x = T/S \quad (25a)$$

$$y = d/T \quad (25b)$$

Equation 25a is the same expression used in the section of the illumination calculation sequence (Eq 7). The diffuse radiation is represented by d. In this application,<sup>3</sup> the total radiation is assumed to be the arithmetic sum of the diffuse and direct radiation,:

$$T = D + d \quad (26)$$

Using these last three expressions we can express the total, direct and diffuse radiation in terms of the atmosphere free radiation.

$$T = xS \quad (27)$$

$$D = \alpha S \quad (28)$$

$$d = (x - \alpha)S \quad (29)$$

These are the same as Eqs 7, 10, and 11 used in the previous section. The direct radiation extinction factor,  $\alpha$ , is expressed in terms of the two ratios (Eq 25):

$$\alpha = x(1 - y) \quad (30)$$

Whenever the weather data include reliable measurements of at least two radiation values (e.g. D, d, or T), the two extinction factors can be directly calculated and the remaining calculation sequence can proceed.

On the other hand, if only one radiation value is available, this paper proposes the other two can be calculated by postulating the existence of a functional relationship between the two ratios, x and y.

#### The Functional Relationship Between x and y

The fraction of the total radiation perceived as diffused is postulated to depend on the total radiation extinction factor in a way that obeys the following criteria:

1. As the extinction factor becomes smaller ( $x \rightarrow 0$ )
  - a. the total radiation is perceived as completely diffuse ( $y \rightarrow 1$ ) and
  - b. the rate at which this perception occurs is gradual and leads to mostly diffuse perception even for  $x \neq 0$ , but  $x \ll 1$ ; i.e.,  $dy/dx \rightarrow 0$  (as  $x \rightarrow 0$ );
2. As the extinction factor becomes larger and nears  $x = 1$ , the atmospheric conditions become progressively more like atmosphere-free conditions:
  - a. the total radiation is perceived completely as direct ( $y \rightarrow 0$ ) and
  - b. the rate at which this perception occurs is gradual; i.e.,  $dy/dx \rightarrow 0$  as  $x \rightarrow 1$ .

Any function that (1) satisfied these criteria and (2) reproduces measured data is a candidate for the model. In this paper, the author uses the function

$$y = 1 - x^n \quad (31)$$

where the exponent n is a function of x, and it is expressed by Eq 9:

$$n = 3.37[1 - e^{-(1-x)}] \quad (32)$$

Equation 31 satisfies the criteria set above:

- 1a.  $y \rightarrow 1$  as  $x \rightarrow 0$
- 1b.  $dy/dx \rightarrow 0$  as  $x \rightarrow 0$
- 2a.  $y \rightarrow 0$  as  $x \rightarrow 1$
- 2b.  $dy/dx \rightarrow 0$  as  $x \rightarrow 1$

The physical interpretation of this function is limited of course to the range  $0 \leq x \leq 1$ .

Figure 4 displays the function (Eq 31) together with the SERI data displayed in the same format. Figure 5 displays the alternative representation implied by Eq 27, 28, and 29, where the total diffuse radiation are expressed as fractions of the atmosphere free value (which is a function of the direct radiation). The SERI measurements are also displayed in the same format.

## LUMINOUS EFFICACY OF THE SKY

The luminous efficacy of the sky was determined by plotting the illumination versus the radiation measurements reported in the SERI data. Measurements were taken simultaneously for total and diffuse radiation on the horizontal plane as well as for total and diffuse illumination, also measured on the horizontal plane.

Figure 6a, shows the total illumination versus the total radiation, while Fig. 6b plots the diffuse illumination versus the diffuse radiation. The resulting plots form almost straight lines, whose slopes are: 33.4 lm h/BTU (114 lm/w) 36.9 lm h/BTU (126 lm/w) for the total diffused efficacy of solar radiation because of diffusion and absorption in the atmosphere. The diffuse efficacy is greater than the total efficacy because the diffuse radiation component experiences greater absorption than does the direct component.

Other authors report similar values for the luminous efficacy. Krochman<sup>4</sup> reports an average value for the overcast sky of 33.7 lm h/BTU (115 lm/w); while Gillette<sup>5,6</sup> reports a total efficacy of 33.2 lm h/BTU (110 lm/w).

The average luminous efficacy obtained from the SERI data breaks down as follows:

	Clear sky (day 68)	Overcast sky (day 62)	Combined (days 62 & 68)
Total	113.10 ± 0.35	116.48 ± 1.10	113.50 ± 0.51
Diffuse	127.53 ± 0.75	124.90 ± 1.10	126.35 ± 0.64
Direct	110.49 ± 0.93	17.13 ± 18.76	109.33 ± 7.64

The direct luminous efficacy was obtained by dividing the direct illumination by the direct radiation. The direct illumination was calculated by taking the difference between the total and diffuse illumination readings. The unreliability of the direct efficacy value of the overcast sky can be understood by studying fig. 5. The diffuse and total radiation are very steep functions of the direct extinction factor in the region of small  $\alpha$ ; thus, a small error in the readings can lead to a large error in the derived calculations.

The key assumption made in calculating the direct efficacy from the SERI data was that the total illumination,  $E$ , is the sum of the diffuse illumination,  $L_d$ , and the direct illumination,  $L_D$ . (An identical assumption in regard to solar radiation.) Thus, in addition to the condition shown in Eq 26, the following condition exists

$$E = L_d + L_D \quad (33)$$

If the efficacies are defined to be  $\epsilon_t = E/T$ ,  $\epsilon_d = L_d/d$  and  $\epsilon_D = L_D/D$  for the total, diffuse, and direct components; it follows that these efficacies should be related to  $y$  as defined by Eq 25:

$$(\epsilon_t - \epsilon_D) / (\epsilon_d - \epsilon_D) = y \quad (34)$$

Using the clear and overcast sky condition values from the preceding SERI data the values of  $y$  are found to be  $(0.15 \pm 0.06)$  and  $(0.92 \pm 0.16)$ , respectively. These values agree with the  $y$  values obtained from the radiation measurement displayed in Figure 1.

These results demonstrate that the calculation sequence and its assumptions are internally consistent. Equation 34 is an expression indicating the relationship between the three luminous efficacy values. It does not, however, show how individual efficacy values are calculated.

The immediate usefulness of these results lies in their justification of total illumination as the product of the total radiation times the total visual efficacy (Eq 23).

## LUMINANCE AND ILLUMINATION

The orientation factors (Eqs 18 and 20) are derived by integrating the sky luminance over the solid angle subtended by a light meter with a "180° field of vision." In general, the sky luminance,  $B$ , is a function of position in the sky, as well as a function of sky conditions. If the position in the sky is defined by the zenith angle,  $\beta$ , and azimuth, angle,  $\phi$ , or by the unit vector,  $\hat{r}$ , then the illumination along the direction defined by the unit vector,  $\hat{n}$ , is expressed as

$$E = \hat{n} \cdot \int \hat{r} B(\beta, \phi) d\Omega \quad (35)$$

where the integration over the solid angle is carried out over the sky dome within the field of view of the measuring device. The functional dependence of the luminance on the sky position is not generally known; however, various formulas are in common use.

### Overcast Sky

There are two formulas commonly used to describe the luminance of an overcast sky, the uniform sky formula and the Moon Spencer formula<sup>7</sup>.

The Uniform Sky Formula. In this model the sky luminance is assumed to be constant. Mathematically it is the simplest to deal with since the function  $B$  is a constant, and it can be taken out of the integral in Eq 35. If the field of view of the measuring device is of a plane, integrating Eq 35 results in the expression

$$E = \pi GB \quad (36)$$

where  $G = \frac{1}{2}(1 + \cos\theta)$  is the sky view factor of a plane inclined at an angle  $\theta$ . If the illumination on a horizontal plane ( $\theta = 0$ ) is  $E_h$ , then the luminance of a uniform sky is

$$B = E_h / \pi \quad (37)$$

The illumination on a vertical plane ( $\theta = 90^\circ$ ),  $E_v$ , is one-half as large as the illumination on the horizontal plane if the ground reflections are ignored.

$$E_v / E_h = 0.5 \quad (38)$$

The Moon Spencer Formula. This formula assigns a zenith angle dependence to the luminance function,  $B$ , while treating the sky as symmetrical in the azimuth direction.

$$B = B_z(1 + 2 \cos\beta) / 3 \quad (39)$$

where  $B_z$  is the luminance in the zenith direction.

Integrating Eq 35, using 39 as the functional dependence of  $B$ , results in the expression

$$E = ME_h \quad (40)$$

where  $M$  is the orientation factor given in Eq 20.  $E_h$  is the illumination on a horizontal plane.

The zenith luminance is, therefore expressed as

$$B_z = \frac{9}{7\pi} E_h \quad (41)$$

The illumination on a vertical plane is about 40% of the illumination on the horizontal plane:

$$E_v = E_h(3\pi + 8) / (14\pi) \approx 0.396E_h \quad (42)$$

Figure 7 shows the ratio of the SERI data taken by the west meter to the measurements taken by the horizontal meter for day 62 (overcast sky). The agreement with the Moon Spencer value (Eq 42) is quite close during the morning hours; but, during the afternoon, the values oscillate around the uniform sky ratio value (Eq 38).

## Clear Sky

A commonly used expression for the clear sky luminance is attributed to Kittler<sup>8</sup>. This same expression is found in the Illumination Engineering Society Handbook<sup>9</sup>. A circumsolar model is being proposed by Littlefair<sup>10</sup>, and a variation of the same idea has been developed separately by this author in the course of the work presented here.

The Kittler Formula. The Kittler formula can be expressed in the form

$$B = B_z f(\delta) \Phi(\beta) / f(\delta_z) \Phi(0)$$

where

$$\begin{aligned} f(p) &= 0.91 + 10 \exp(-3p) + 0.45 \cos^2(p) \\ \Phi(p) &= 1.0 - \exp(-0.32/\sigma_z); \Phi(0) = 0.274 \\ \delta &= \text{angular distance from the sun to the given point in the sky} \end{aligned}$$

The illumination is obtained, as in the other cases, by integrating Eq 35. In this case, however, a numerical integration is required. Where the results of this integration were compared with the SERI measurements the agreement was very poor.

A separate calculation was carried out by Littlefair<sup>11</sup>, and his results showed a much improved agreement. The reason for the inconsistency between the two sets of calculation had not been resolved as of this writing; therefore, no further elaboration on this formula will be presented.

The Circumsolar Model. The circumsolar model as conceived by this author assumes that the clear day luminance should collapse into a Dirac delta function in the limit as the atmosphere disappears. Using the atmosphere free limit for the value of the luminance function,

$$B(\beta, \phi) = A \delta(\beta - \beta_s) \delta(\phi - \phi_s) \quad (44)$$

where  $\beta$  is the zenith angle and  $\phi$  the azimuth. The subscript s represents the solar value.

Using this expression in Eq 35, the expression for the illumination becomes

$$E = (A \sin \beta_s) \sigma_n \quad (45)$$

This expression contains the orientation factor,  $\sigma_n$  (Eq 18), used in the illumination calculation sequence. Although the orientation factor,  $\sigma_n$ , yields excellent agreement with the measured values of illumination on a plane with a 180 field view on unobstructed surfaces, it is not useful in calculating the illumination of an object with a smaller field of view. If the solid angle vector does not point directly into the sun, Eq 44 will yield for the illumination a value equal to zero. Work on the development of this model continues as of this writing.

## THE EMPIRICAL DATA

The empirical data were made available to this author through the courtesy of the Solar Energy Research Institute. The data received comprised two full days:

Julian day 62 (March 3), 1981; overcast sky

Julian day 69 (March 9), 1981; clear sky

The data were collected at irregular intervals. The instruments appeared to have been well calibrated. Only the data points collected when the sun was low in the sky appeared to be unreliable. The total number of readings used in this analysis was 78 for the overcast day and 110 for the clear day. The reported latitude of the experimental site is 40°N, and the data are consistent with this latitude. The local time was reported to be the true solar time; the measured values, however, indicate that the local time differs from the solar time by about seven minutes. The measurements were recorded in the presence of a painted egg crate to reduce the ground reflectance. Upon comparing the calculated illumination values with the measured values, a systematic difference was found

that could be corrected by assigning a finite reflectance to the ground. Using Eq 24 with a background reflectance of  $r = 6\%$  corrected this systematic difference. These observations were reported to the SERI personnel and were verbally confirmed by them<sup>12</sup>.

The data from the following recording channels were used:

Channel	Description
900	Total horizontal illumination
901	Diffuse horizontal illumination
902	Total vertical illumination (North)
904	Total vertical illumination (East)
906	Total vertical illumination (South)
908	Total vertical illumination (West)
911	Total horizontal radiation
912	Diffuse horizontal radiation

The data were recorded in millivolts with the following calibration factors: Illumination  $1 \text{ mV} = 10^4 \text{ lx}$ ; radiation  $1 \text{ mV} = 10^2 \text{ W/m}^2$ . The diffuse values need to be corrected by the band shadow factor of 1.12. The diffuse radiation needs to be multiplied by the spectral shift factor,  $\beta = 1/[1.17 - 1/(1.2 + 11.8p)]$

where  $p$  = ratio of readings from channel 912 to 911.

#### NOMENCLATURE

B	Luminance (lux/steradian) foot candle/sr ( $[lx/sr]$ )
$B_z$	Zenith luminance
D	Julian day
D	Direct solar radiation $\text{BTU/h ft}^2$ ( $[\text{W/m}^2]$ )
$D_{\text{max}}$	Direct normal radiation
$D(\hat{n})$	Direct radiation in the direction $n$
d	Diffuse solar radiation
$d_{\text{max}}$	Diffuse zenith solar radiation
$d(\hat{n})$	Diffuse solar radiation in the direction $n$
E	Total illumination foot candle ( $[lx]$ )
$E_h$	Horizontal total illumination
$E_v$	Vertical total illumination
G	Sky view factor
H	Hour angle
h	Hour of day (solar time)
$L_D$	Direct illumination
$L_d$	Diffuse illumination
M	Diffuse radiation orientation factor
n	Dimensionless exponent
$\hat{n}$	Orientation unit vector
$n_s, n_e, n_z$	South, east, and zenith components of $\hat{n}$
$\hat{r}$	Position unit vector
S	Atmosphere free solar radiation on horizontal plane
$S_c$	Solar constant

x	Total radiation extinction factor
y	Ratio of diffuse to total radiation
$\alpha$	Direct radiation extinction factor
$\beta$	Zenith angle
$\beta_z$	Zenith solar angle
$\delta$	Angular distance from the sun
e	Luminous efficacy
$\phi$	Azimuth angle
$\phi_s$	Azimuth solar angle
$\gamma$	Solar declination
$\Omega$	Solid angle
$\hat{\sigma}$	Unit vector of the sun's position
$\sigma_s, \sigma_e, \sigma_z$	South, east, and zenith components of $\hat{\sigma}$
$\sigma_n$	Component of $\hat{\sigma}$ along $\hat{n}$
$\theta$	Zenith angle of observation plane

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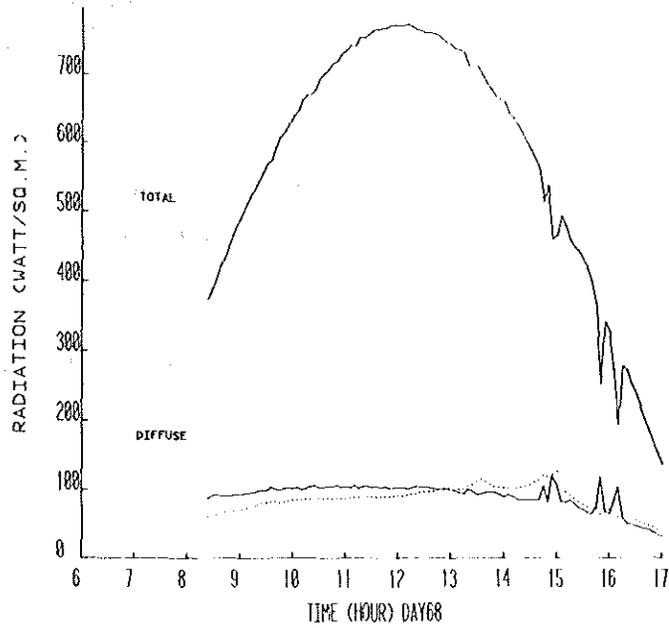
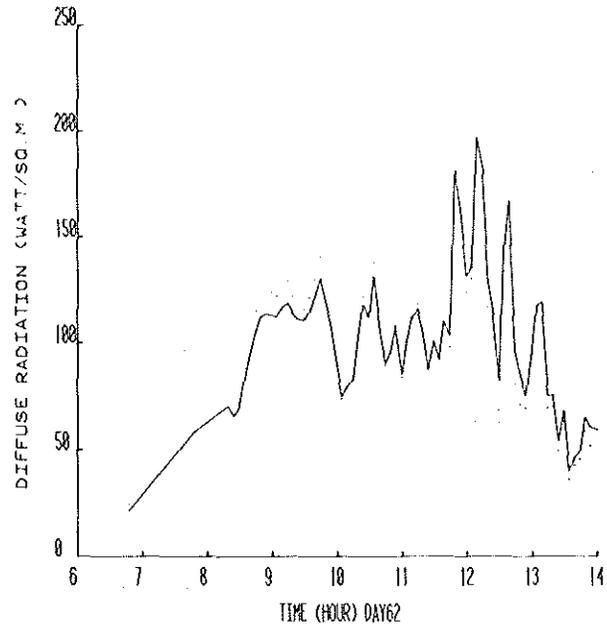


Figure 1. Radiation on a horizontal plane. Total and diffuse shown for overcast sky and clear sky conditions. Dots represent the SERI measurements. Calculated values from Eq 11 in solid lines.

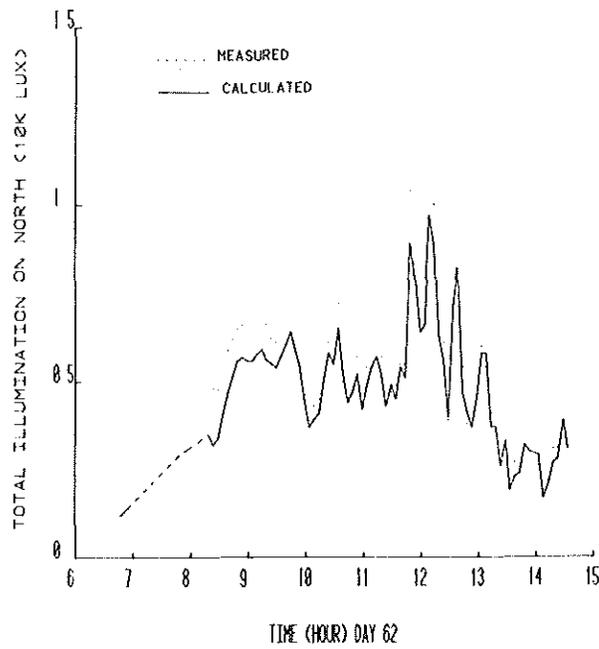
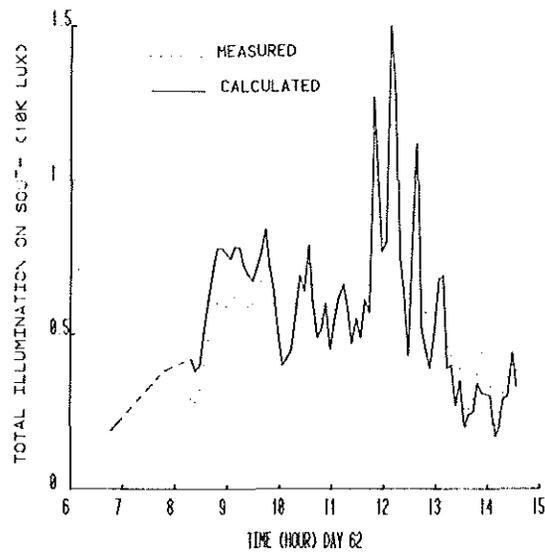


Figure 2. Overcast sky. Illumination along two of the cardinal orientations. SERI data shown in dots. Calculated values (Eq 23) shown in solid curves.

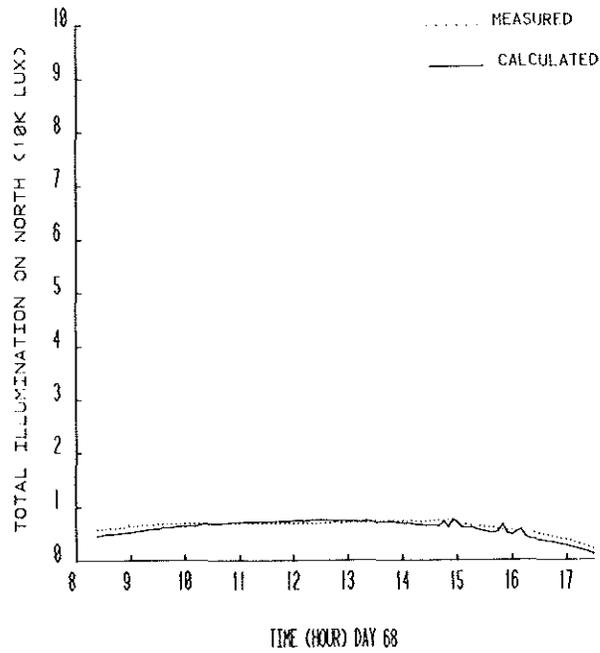
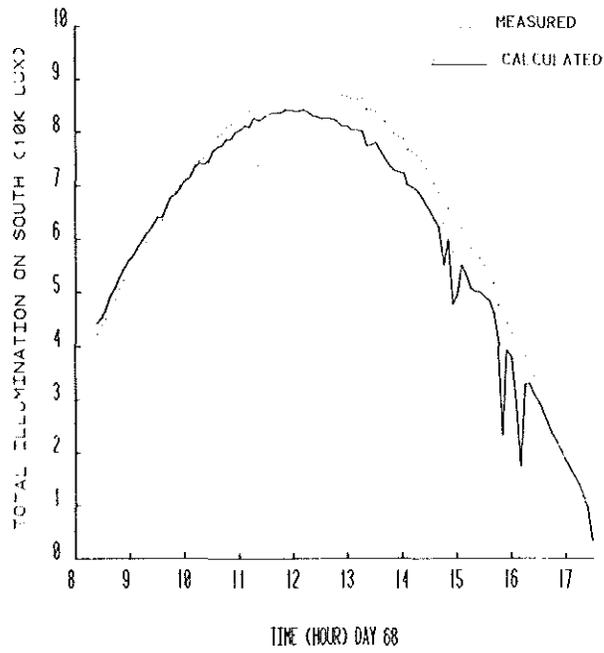


Figure 3. Clear sky. Illumination along the four cardinal orientations. SERI data shown in dots and Eq 23 shown in solid curves.

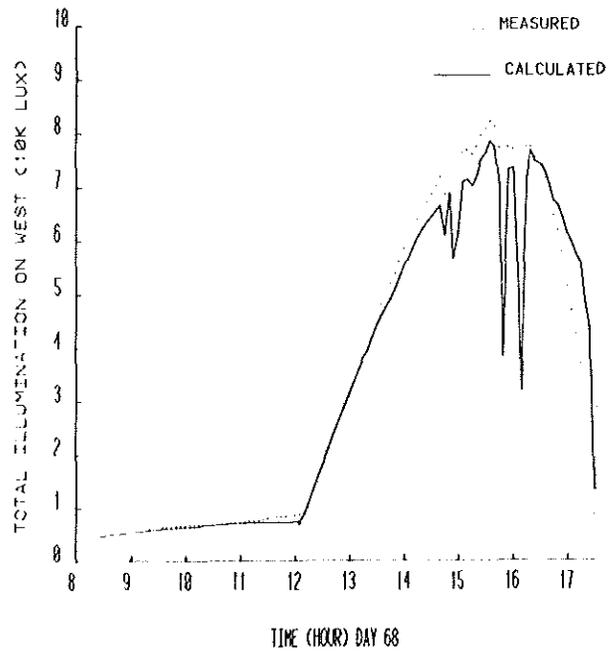
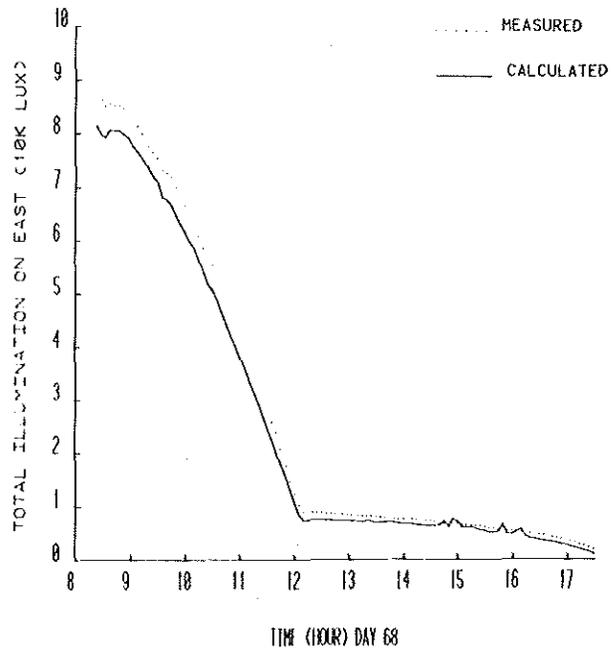


Figure 4. Liu-Jordan Parameters as calculated from the SERI data (dots) and Eq 31 (dashed curve).

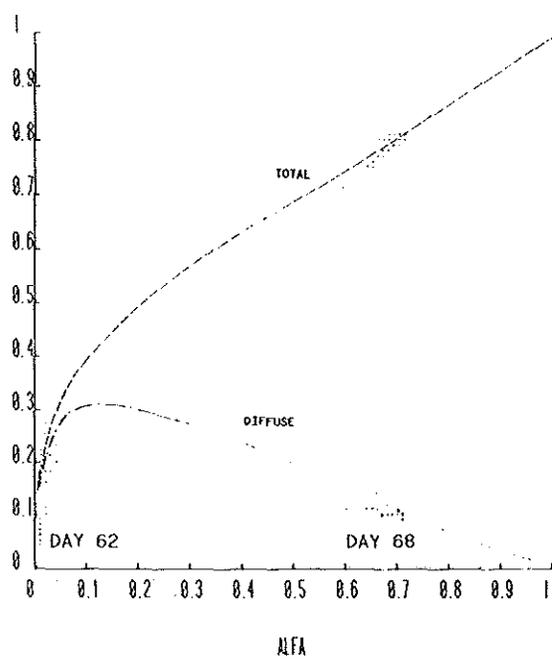
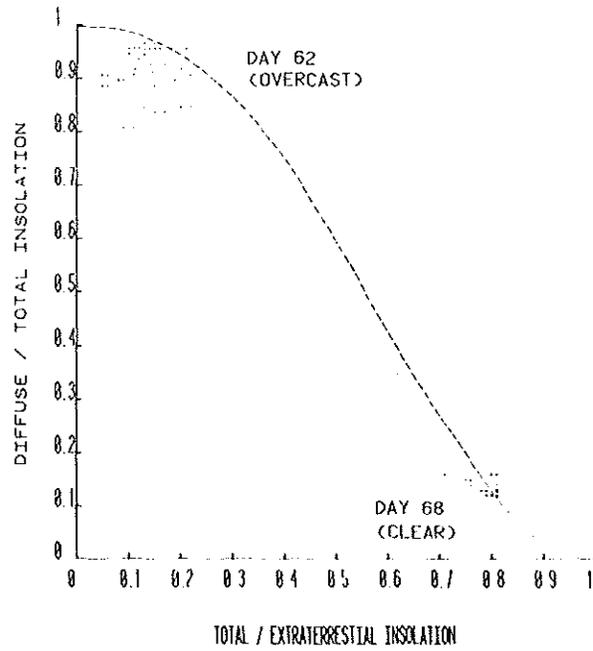


Figure 5. Total, diffuse, and direct solar radiation as a function of  $\alpha$ . SERI data shown with dots. Theoretical values from Eq 27 and Eq 29.

## Discussion

R. Crenshaw, Lawrence Berkeley Laboratory, University of California, Berkeley: How well does approximation work on a monthly basis? Is it consistent across months?

F. Arumi-Noe: I have tested the model only against microclimatic data collected on experimental sites. The tests have been done only for a few days at a time. No annual-length tests have been carried out.

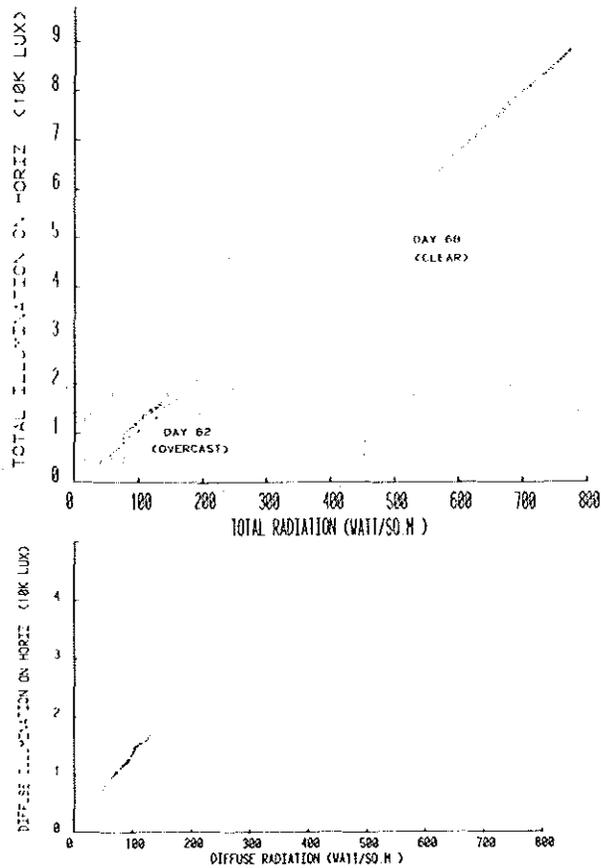


Figure 6. Sky illumination correlated to solar radiation from the SERI data. Total illumination versus total radiation shown in upper graph yield an efficacy (slope) of 114 lumen per watt. Diffuse illumination versus diffuse radiation shown in lower graph yield an efficacy of 126 lumen per watt. Overcast sky and clear sky conditions can be differentiated in the total illumination graph, but not in the diffuse illumination one.

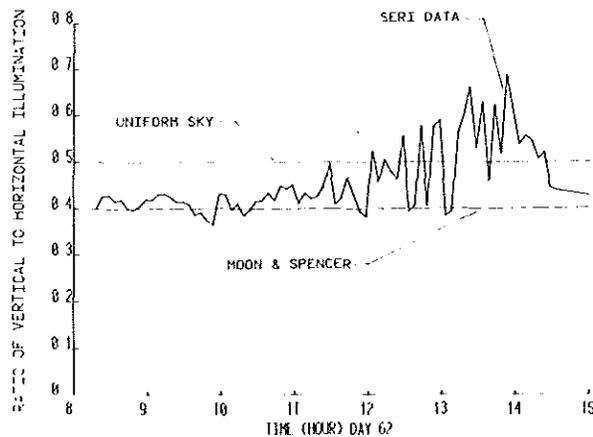


Figure 7. Ratio of vertical to horizontal illumination. Comparison of the Moon and Spencer constant value of .4 (Eq 43), uniform sky value of .5 (Eq 40), and the SERI data.